EFFICIENTLY SOLVING CONSTRAINT OPTIMIZATION PROBLEMS USING LEARNING-BASED TECHNIQUES

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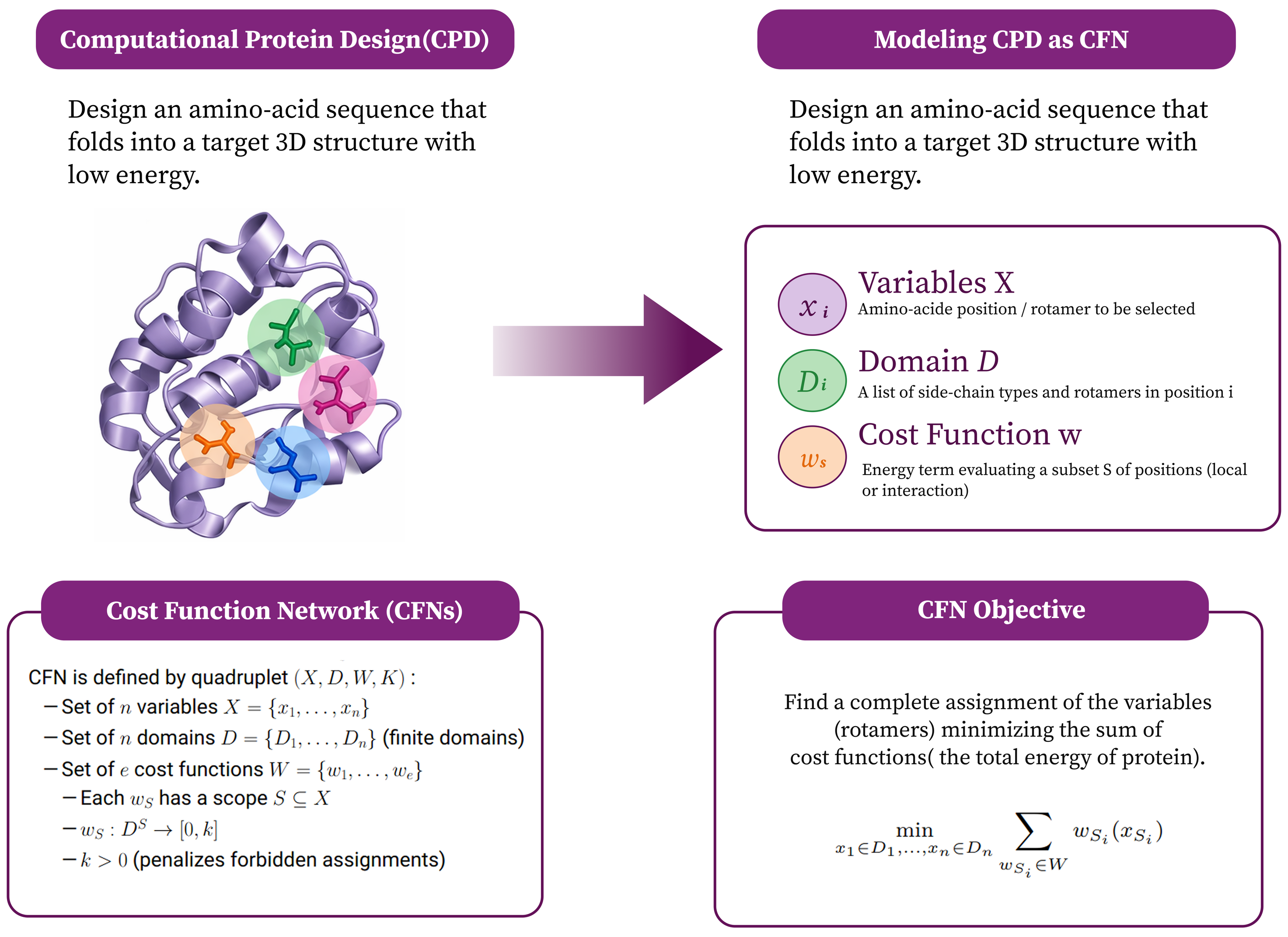
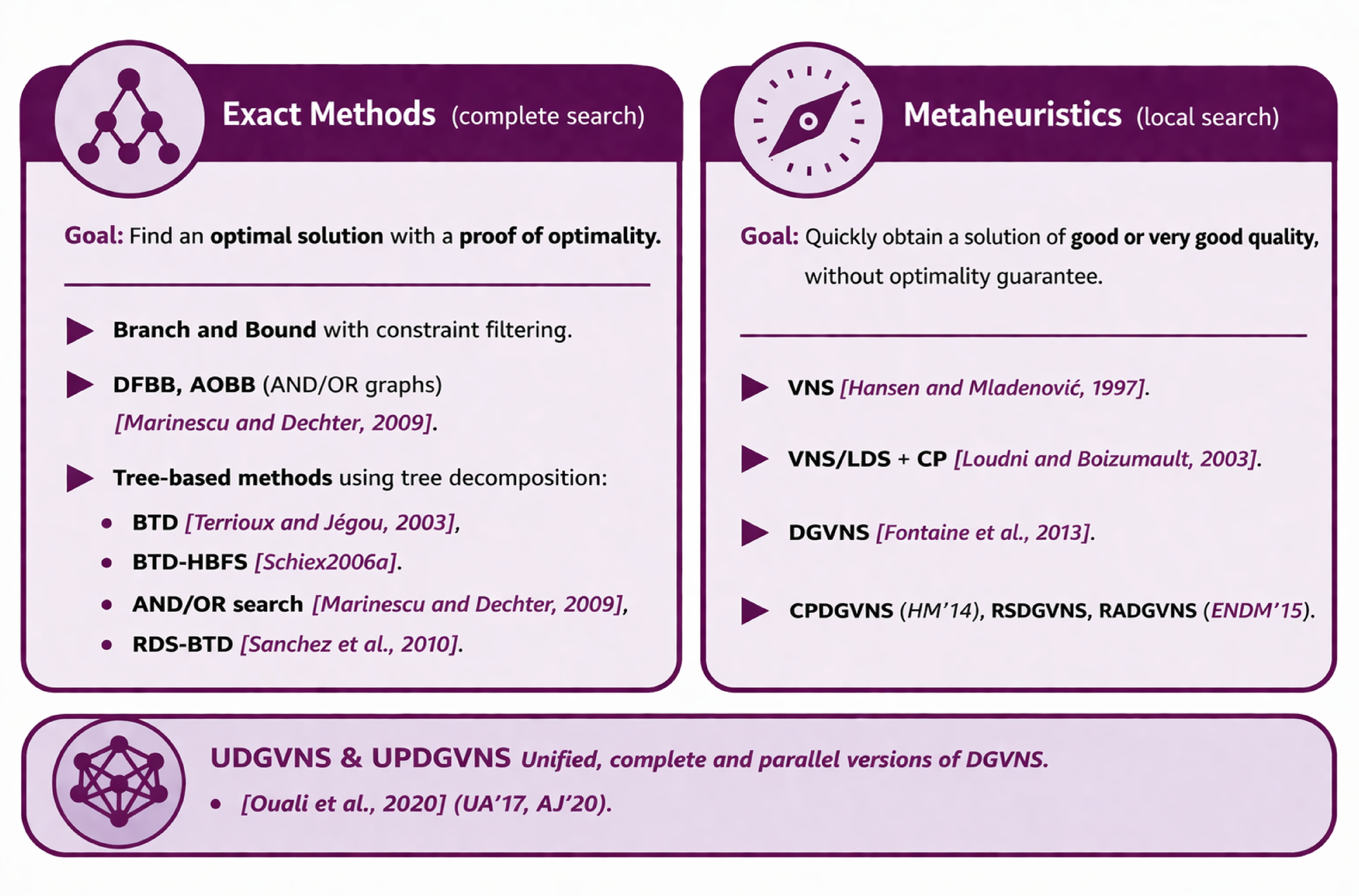
1. Context

Cost Function Networks are a specific type of graphical model used to solve optimization problems in which constraints have associated with a violation costs, where the objective is to find solution that minimizes the final cost.



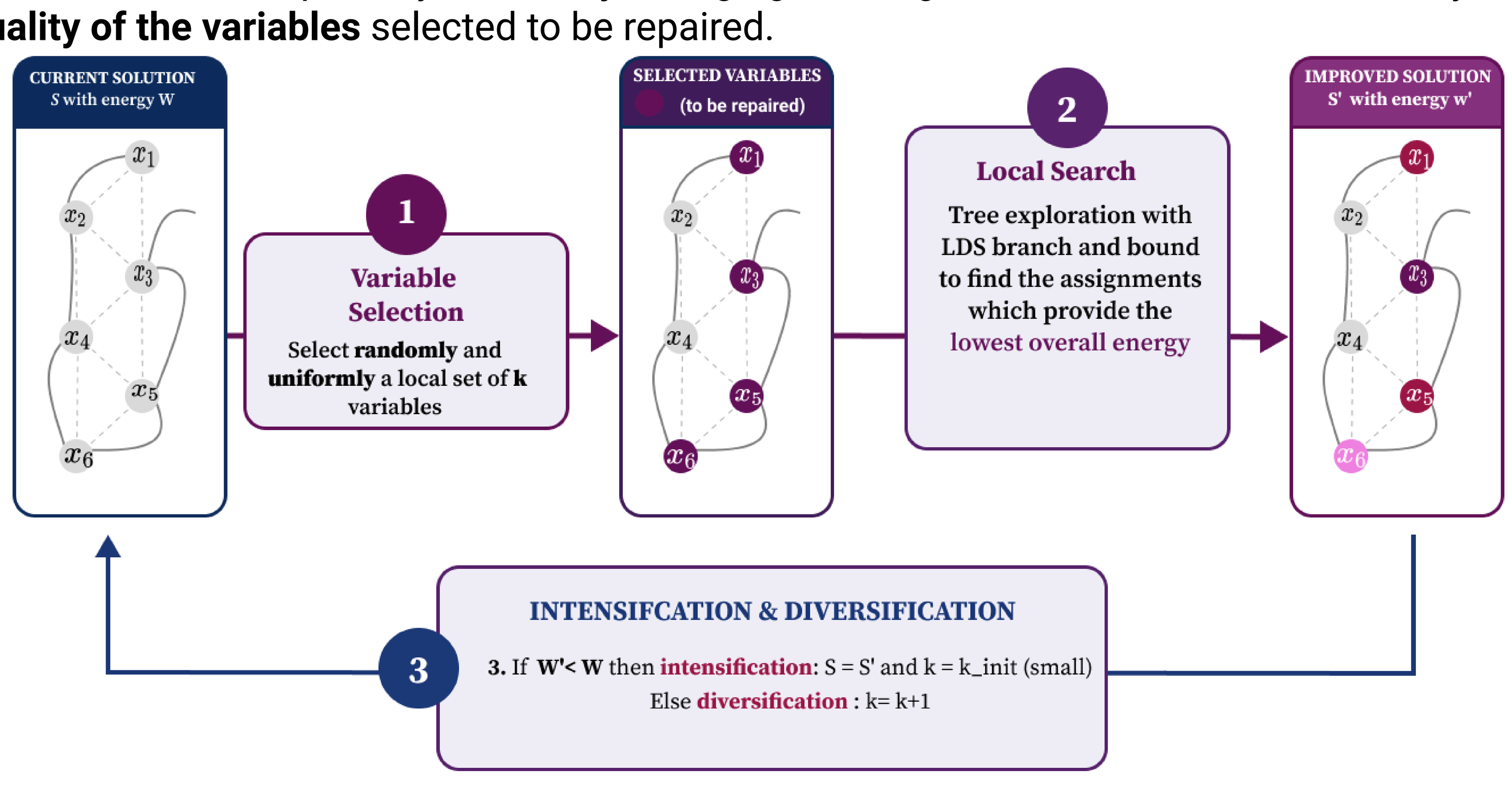
2. Methods for Solving CFNs

CFNs can be addressed by complementary solving strategies, balancing optimality guarantees, scalability, and structural exploitation.



3. VNS/LDS+CP Method

VNS explores the search space by **iteratively** changing the neighborhood size K. Its efficiency relies on the **quality of the variables** selected to be repaired.

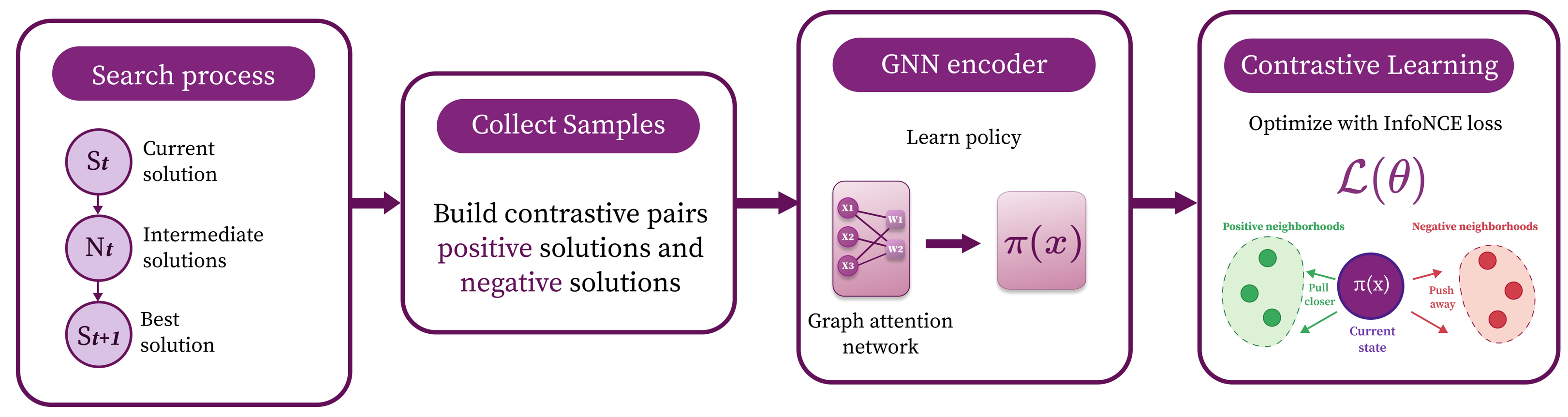


Neighborhood = Combinatorial subspace associated with the Selected variables.

- **Limitations :**
  - Random heuristic without exploiting past search experience
  - Insensitive to variable importance, leading to inefficient repairs
  - Limited in guiding intensification, especially on complex instances
- **Motivation :**
  - Learning from the search history using Contrastive Learning with GNNs
  - Learn a strategy to identify and select better neighborhoods.

4. Contrastive Learning Framework

The model learns a policy where improving variables are pulled closer to the current search state, while ineffective variables are pushed away.

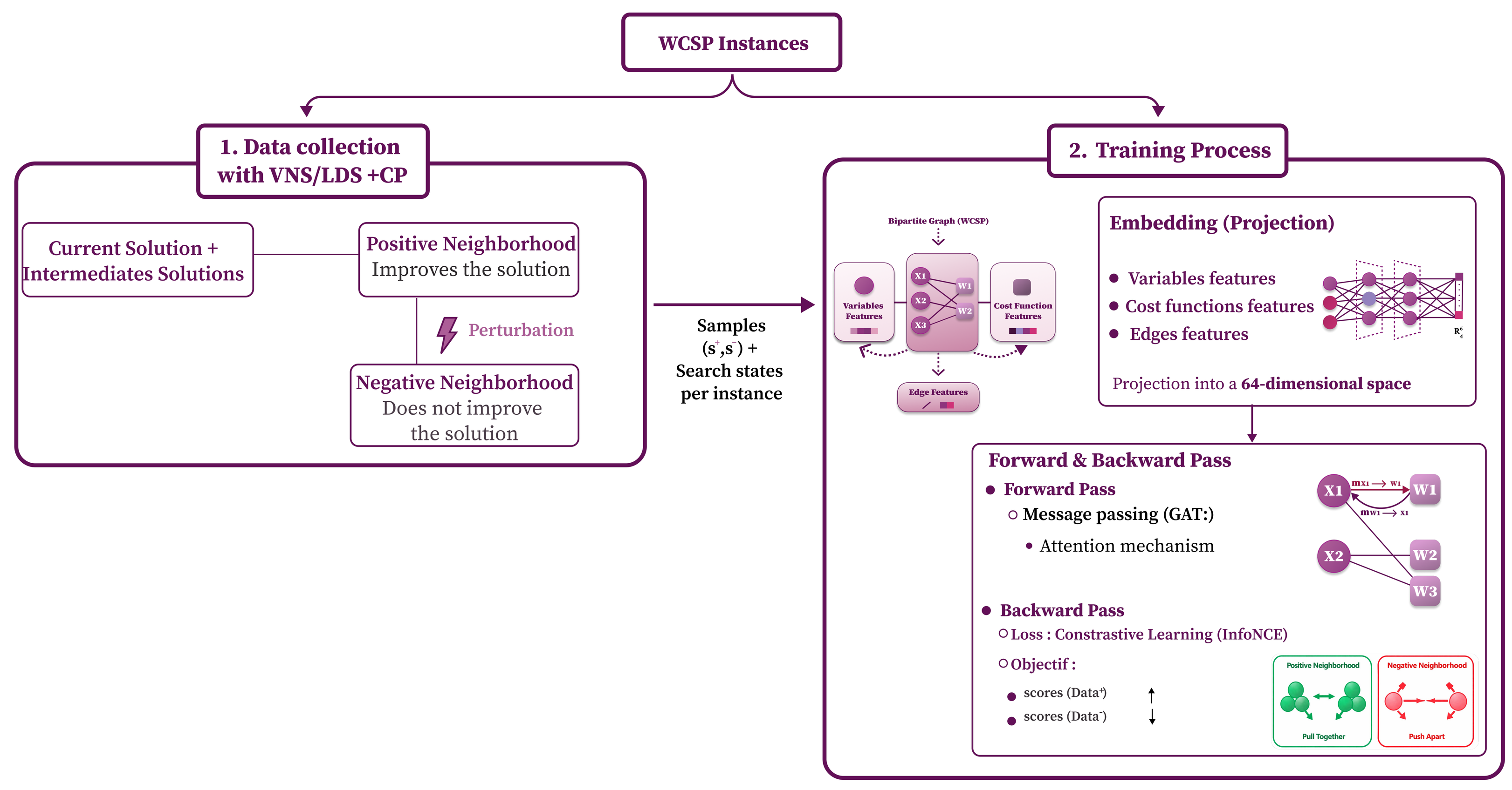


Training loss : InfoNCE encourages similarity with improving variables and dissimilarity with non-improving ones

$$\mathcal{L}(\theta) = \sum_{(s, S_{pos}, S_{neg}) \in \mathcal{D}} \frac{-1}{|S_{pos}|} \sum_{a \in S_{pos}} \log \frac{\exp(a^\top \pi(s)/\tau)}{\sum_{a' \in S_{neg} \cup \{a\}} \exp(a'^\top \pi(s)/\tau)}$$

5. Proposed method CL-VNS

1. Generate training data by using VNS/LDS+CP on CFN instances
   - **Positive solutions :** solutions that approach the best known solution and yield a significant cost gain
   - **Negative solutions :** solutions perturbed from the neighborhood of the best solution that yield only a marginal cost gain
2. Train GNN with supervised contrastive learning to distinguish effective from ineffective variables and guide the search.
   - Attention mechanism on bipartite graph (GAT)
   - Embedding representation using MLP from variable/constraint features.
3. Use the learned policy to predict scores for variables
4. Perform weighted random sampling without replacement to select k variables of the neighborhood



6. Conclusion

- The anytime behavior of VNS/LDS+CP is adequate to collect positive solutions
- CL-VNS capture the current state of the search process to evaluate the importance of variables for the next iteration.
- **Improvements :**
  - Experimenting on larger and more diverse CFN instances
  - Extend the current CL-VNS approach to UDGVNS in order to exploit tree-based decomposition.

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