MSc Internship proposal: Can Deep Learning reveal Ito drift processes in upper ocean dynamics?

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Keywords: upper ocean dynamics, impact of fine-scale processes, data-driven approaches, stochastic calculus, neural networks.

Context: The understanding and evaluation of the impacts of fine-scale random processes onto larger-scale processes is in a key challenge in in physical oceanography. Ito-Wentzell formula provides the basic background to investigate these issues from a theoretical and computational point of view. It states that, for any Ito-drift time process X_t governed by

$$dX_t = \mu_t dt + G_t dB_t$$

with μ_t the drift, B_t a multivariate Wiener process and G_t a matrix, any function f of process X_t is also an Ito-drift process governed by the following stochastic differential equation

$$df(t, X_t) = \left\{ \frac{\partial f}{\partial t} + \langle \mu_t, \nabla f \rangle + \frac{1}{2} Tr \left[G_t^T H_f G_t \right] \right\} dt + \nabla f^T G_t dB_t$$

where H_f is the Hessian of f. When studying upper ocean dynamics, X_t typically refers to state variables where $G_t dB_t$ a fine-scale unresolved random fluctuation, and $f(t, X_t)$ to some observed geophysical tracer. The above equation exhibits a diffusion term $G_t^T H_f G_t$ on the observed tracer caused by the random fluctuation. We may also stress that the numerical resolution of such Ito-drift time processes may involve additional terms depending on the gradient of the random fluctuation. **A key scientific question is to investigate the extent**

to which upper ocean dynamics exhibit such relationships between large-scale processes and fine-scale turbulence processes.

This internship is proposed in the framework of ANR Melody (Bridging geophysics and MachinE Learning for the modeling, simulation and reconstruction of Ocean DYnamics, PI: R. Fablet) and ERC STUOD (Stochastic Transport in Upper Ocean Dynamics, PI: B. Chapron).

Proposed approach: Neural Network schemes, termed neural ODE [2], have recently emerged as new means to analyse, implement and identify ODEs and PDEs (Ordinary/Partial Differential Equation). These NN representations open new avenues for the identification of differential operators from observation and/or simulation data [1, 4, 3].

The goal of this internship will be to investigate how NN schemes could help revealing and understanding from data large-scale drift and diffusion processes caused by fine-scale processes in the upper ocean. For different case-studies, experiments will be performed on numerical simulations (e.g., toy models, reduced ocean models, HR ocean simulations). Experiments on real observation datasets would also be of interest in a second step.

Skills: Msc./Eng. degree in Applied Math., Data Science and/or Physical Oceanography with a good background in applied statistics. Knowledge on deep learning models and experience in deep learning frameowrks (eg, tensorflow, keras, pytorch) would be a plus.

References

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